

Dynamics of a Quadrotor Undergoing Impact with a Wall

Fiona Chui, Gareth Dicker, and Inna Sharf

Abstract—In this paper, we investigate the problem of the dynamics of a quadrotor unmanned aerial vehicle undergoing impact with its environment. This work is motivated by the fact that operation of UAVs (manual or autonomous) carries with it a significant risk of collision with surrounding objects, particularly in unknown, unstructured environments. To make small UAVs more viable and expand their autonomy, our ultimate objective is to develop control methods which would allow automatic recovery from a ‘non-destructive’ collision, where operation of the vehicle is not compromised. Towards this goal, we formulate the dynamics model of a quadrotor equipped with protective bumpers around its propellers, undergoing an arbitrary collision with a vertical wall: no prior assumptions are made regarding the points and number of impacts, nor the impact speed, nor the orientation of the vehicle at the instance of collision. The model is exercised through a series of simulations for different pre-impact attitudes of the platform and different approach speeds. Results of experimental tests conducted with Spiri quadrotor platform are presented. Comparison to the simulated responses mimicking the experimental pre-impact conditions and command inputs show excellent qualitative agreement.

I. INTRODUCTION

A. Background and motivation

In recent years, a substantial amount of research has focused on stable control of Unmanned Aerial Vehicles (UAVs), specifically in the quadrotor configuration. These developments have given rise to their potential use in not only civil and military applications, but as hobbyist and amateur platforms as well [1]. Along with a rise in commercial availability and personal usage, requirements for operational safety of these vehicles in unstructured, ‘out-of-the-lab’ environments have also become more stringent. In response to this demand, more recent designs of quadrotor platforms have emerged which incorporate bumpers (otherwise known as ribbons, shrouds, airframes, or protective frames) that can protect the fragile propellers of the vehicle in the event of a collision, and increase the comfort and safety of humans in the proximity. Examples of such platforms include the Parrot AR Drone 2.0, the UDI U818A, and Spiri from Pleiades Robotics Inc. While the bumpers protect the quadrotor during impact, they do not prevent the vehicle from losing flight control and crashing to the ground, which presents a serious risk for the quadrotor, objects on the ground, and most importantly, to humans. The ability to regain stable control after a collision would increase safety, and potentially lead to greater acceptance of UAVs among government regulators and ultimately the greater public.

All authors are with the Department of Mechanical Engineering, McGill University, Montreal, QC, Canada. {fiona.chui, gareth.dicker}@mail.mcgill.ca, inna.sharf@mcgill.ca



Fig. 1. Spiri experimental platform with protective bumpers [2].

Mid-air impacts with a stationary surface are a common hazard when flying the quadrotor both indoors and outdoors, piloted manually or autonomously. Example situations include a beginner pilot failing to control the quadrotor and crashing it into a wall, a pilot losing perspective of distance between the quadrotor and an obstacle during manual flight control, and flying in a disaster reconnaissance environment with unpredictable debris and rubble. Although obstacle avoidance algorithms have been developed to avoid collisions [3–5], our focus will be on inevitable impacts between a quadrotor and a stationary surface.

Recent work in quadrotor aerobatics under external motion tracking [6, 7] and automatic recovery from arbitrary initial conditions using only on-board sensing [8] show promise for the development of collision recovery control in general. Although the quadrotor maneuvers in these works occur in unconstrained space, they nevertheless provide a good starting point for the aerobatic maneuvers needed for collision recovery control. Toward this goal, we have developed a simulator which incorporates contact dynamics into the quadrotor rigid-body dynamics model to allow us to simulate quadrotor response in a collision. This represents a crucial step to developing state estimation and control algorithms to successfully recover control of a quadrotor aircraft following a non-destructive impact with its environment, using only on-board sensors. In the present paper, collision is modelled with a flat vertical wall, a surface which serves as a good representation of many indoor and outdoor collision hazards and also allows for reasonable and experimentally reproducible testing and validation in the lab.

Through an accurate contact dynamics model, characterization of the post-impact responses under a range of incoming impact velocities and attitudes will be possible, making available novel information on rebound direction and spin that will be used to develop recovery manoeuvres for any initial impact conditions that translate to a non-destructive collision.

B. Related work

The non-linear dynamics of a quadrotor has been modelled to varying degrees of accuracy, depending on the control application. The basic model consists of a Newton-Euler formulation of the equations of motion for a rigid body, with the moments and forces due to thrusts applied at the rotor locations. Additional dynamic effects in this model include the gyroscopic torque [9, 10], the ground effect [10], and blade flapping during translational flight [11]. The aerodynamic effect of airflow disruption from bumpers in close proximity to the propeller was found to be significant and resulted in yaw tracking inconsistencies, but was not modelled in [11].

Changes in UAV flight dynamics due to interactions with the environment have been studied by several authors. Stabilizing controllers for a single UAV with dexterous manipulator attachments for grasping objects have been developed [12–15], including a helicopter controller that makes use of a spring contact model at an end-effector located below the helicopter body [15].

A hybrid controller for a robot with flying-walking locomotion was developed by using a dynamics model that includes contact at the two supporting feet, represented as spherical joints [16]. When in the continuous contact mode, a supporting force is applied on the stationary foot, while in the discrete impact mode, impacts between the ground and foot are assumed inelastic. For both modes, friction at the ground prevents sliding motion and external moments are not generated by contact at the ground [16].

To date, there are no published works dealing specifically with the impact scenario between a quadrotor and a stationary object, as would result from an accidental collision, although, several papers address the scenario of a quadrotor purposefully interacting with the environment. In [17, 18], a model and controller for a Ducted-Fan Miniature UAV (DFMAV) interacting with a fixed vertical surface are presented. The impact force at a specified point of contact on the body was calculated using the linear Kevin-Voigt model. The controller uses a hybrid automaton to manoeuvre the DFMAV from free-flight to dock with the wall at the contact point, slide along the wall, and undock to free-flight. The simulation is able to capture the undesired ‘rebound’ dynamics of the DFMAV that may occur when trying to dock from free-flight. Another hybrid automaton controller of note allows for a quadrotor with a wire airframe to perform docking and sliding manoeuvres on walls at specified contact points [19]. During its development, it was found that a contact point above the quadrotor’s center of gravity will reduce external moments and the tendency for the quadrotor to flip during docking. The collision was also modelled using the Kevin-Voigt model, while the sliding motion resistance was modelled with viscous friction. In experiments, collision was detected with force sensors at the docking points and the controller successfully minimized rebounds due to impact before entering the sliding mode.

The maturity of research on contact dynamics modelling and solution algorithms was leveraged for developing our

dynamics model. There exists a rich body of knowledge in the fundamentals of contact mechanics and different contact models, with many diverse applications including robotics [20, 21], biomechanics [22, 23], automotive vehicles [24, 25], and now quadrotors. A review and ranking of continuous non-linear contact models was provided in [26], along with an introduction and comparison of a new contact model. Together with a survey of contact dynamics modelling [27], these works provided a basis for choosing the model for studying quadrotor response undergoing impact with a wall.

C. Contributions

We present a continuous model of a quadrotor platform with protective bumpers around its propellers which captures the response after a non-destructive impact with a flat vertical wall. The contact model is more general compared to other models that describe UAV interaction with walls in that the contact point is not defined beforehand and the contact force relationship is non-linear. The simulated aircraft is based on our experimental platform: the Spiri quadrotor in Figure 1. The quadrotor response to different initial impact conditions is examined, and is validated with experimental data.

In Section II, we provide the Newton-Euler formulation of the quadrotor’s rigid-body dynamics equations, which includes contact forces and corresponding moments. Our contact model and definition of contact geometry used for finding these forces and moments are presented in Section III. Simulation results for different impact scenarios are included in Section IV, with experimental validation in V.

II. DYNAMICS MODEL

Both the inertial frame $\mathcal{F}_I = \{\mathbf{e}_x \ \mathbf{e}_y \ \mathbf{e}_z\}$ and body-fixed (quadrotor) frame $\mathcal{F}_Q = \{\mathbf{e}_x \ \mathbf{e}_y \ \mathbf{e}_z\}$ are required to describe the full dynamics and kinematics of the quadrotor. The body frame with origin at the vehicle center of mass (CM) is defined such that $\mathbf{e}_x$ points outwards from the front of the vehicle body, $\mathbf{e}_z$ points downwards, and $\mathbf{e}_y$ is chosen to follow the right-hand rule. The dynamics of our quadrotor are modelled using the Newton-Euler formulation for a rigid body:

$$m\dot{\mathbf{v}} + m\boldsymbol{\omega} \times \mathbf{v} = \mathbf{F}_G + \mathbf{F}_T + \mathbf{F}_C \quad (1)$$

$$\mathbf{I}\dot{\boldsymbol{\omega}} = \left(\sum_{j=1}^4 \mathbf{r}_{Tj} \times \mathbf{F}_T \right) + \mathbf{M}_T + \mathbf{M}_\Omega + \mathbf{M}_C - \boldsymbol{\omega} \times \mathbf{I}\boldsymbol{\omega} \quad (2)$$

and its pose kinematics are propagated using the quadrotor velocities with:

$$\dot{\mathbf{p}} = \mathbf{R}^T \mathbf{v} \quad (3)$$

$$\dot{\mathbf{q}} = -\frac{1}{2} \begin{bmatrix} 0 \\ \boldsymbol{\omega} \end{bmatrix} \otimes \mathbf{q} \quad (4)$$

where $\mathbf{v} = [u \ v \ w]^T$ and $\boldsymbol{\omega} = [p \ q \ r]^T$ are the linear and angular velocities expressed in $\mathcal{F}_Q$, and $\mathbf{p}$ is the position of the quadrotor CM in $\mathcal{F}_I$. The platform specific parameters m , $\mathbf{I}$, and $\mathbf{r}_{Tj}$ are the mass, moment of inertia matrix, and

relative position of the thruster locations to the CM respectively, where j is the propeller/bumper index. The quaternion $\mathbf{q} = [q_0 \ q_1 \ q_2 \ q_3]^T$ describes the orientation of the quadrotor with respect to $\mathcal{F}_I$, and $\mathbf{R}$ is the rotation matrix which transforms a vector from $\mathcal{F}_I$ to $\mathcal{F}_Q$. A quaternion formulation was chosen to parametrize the quadrotor orientation due to the aerobatic nature of the post-impact response. The cross product and quaternion multiplication operators are denoted by $\times$ and $\otimes$, respectively.

The applied forces and moments $\mathbf{F}$ and $\mathbf{M}$ are components in $\mathcal{F}_Q$, where the subscripts G , T , Ω , and C denote gravitational, thruster, gyroscopic, and contact respectively. They are formulated in (5) to (7) below, with the exception of $\mathbf{F}_C$ and $\mathbf{M}_C$, which will be derived in Section III. Forces and moments due to aerodynamic drag and propeller flapping are neglected.

$$\mathbf{F}_G = \mathbf{R} \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix}, \quad \mathbf{F}_T = \begin{bmatrix} 0 \\ 0 \\ -k_t(\sum_{j=1}^4 \Omega_j^2) \end{bmatrix} \quad (5)$$

$$\mathbf{M}_T = \begin{bmatrix} 0 \\ 0 \\ d_t \sum_{j=1}^4 (-1)^j \Omega_j^2 - J_r \sum_{j=1}^4 (-1)^j \dot{\Omega}_j \end{bmatrix} \quad (6)$$

$$\mathbf{M}_\Omega = \begin{bmatrix} -q J_r \sum_{j=1}^4 (-1)^j \Omega_j \\ p J_r \sum_{j=1}^4 (-1)^j \Omega_j \\ 0 \end{bmatrix} \quad (7)$$

The propeller lumped thrust and drag torque coefficients are k_t and d_t respectively, and J_r is the propeller moment of inertia about its rotational axis. The dynamics are driven by the four propeller angular speeds Ω_j , which serve as the control inputs generated by a typical quadrotor attitude controller. The angular acceleration $\dot{\Omega}_j$ is simply the derivative of the corresponding propeller angular speed control input. Many dynamics models in literature ignore the gyroscopic moment (7) and the angular acceleration term in (6), since their effects are cancelled from counter-rotating propeller pairs when yaw is stable [11] and in equilibrium hovering conditions. We have found these terms important in the aerobatic post-impact responses of the vehicle and therefore, retain these propeller-dynamics induced terms in our model.

III. CONTACT MODEL

The contact forces and moments are a result of the impact force applied at the point of collision, a.k.a., the contact point, as determined by the contact geometry.

A. Contact force model

The force F_n applied at the contact point $\mathbf{p}_C$, in the direction normal to the contact surface is modelled using a non-linear compliant model originally introduced by Hunt and Crossley [28]:

$$F_n = \lambda \delta^n \dot{\delta} + k \delta^n \quad (8)$$

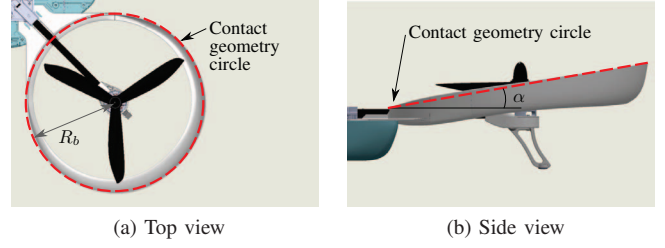


Fig. 2. Basis for contact geometry from experimental platform

where δ is the local deformation (penetration) at $\mathbf{p}_C$, k is a constant stiffness coefficient, λ is a damping coefficient, and n is dependent on the contact scenario. The Herbert and McWhannell model in (9) for relating λ to the coefficient of restitution e is used here because of its accuracy over the other proposed approximations [26, 29]. The corresponding relationship is:

$$\lambda = \frac{6(1-e)}{[(2e-1)^2 + 3]} \frac{k}{v_i} \quad (9)$$

where v_i is the initial impact velocity of $\mathbf{p}_C$, that is, $\dot{\delta}$ at the beginning of collision.

B. Contact geometry

The 3-D printed nylon bumpers on our experimental platform are individually attached to the central body of Spiri at a slight angle, and are approximately circular on the outside edges: they are most likely to contact the wall first in a collision. Our contact geometry model is a simplified representation of this configuration, and is comprised of four circles with a radius R_b , each centered on the propeller axis, and tilted at an angle α towards the body center, as shown in Figure 2. It is noted that the circular contact geometry, aside from representing the bumper geometry closely, would have to be used even in the absence of bumpers in order to detect collisions with a propeller, the tip of which traces a circular path.

An algorithm was formulated to determine the point of contact $\mathbf{p}_C$ between a circle and a vertical wall, from which the penetration distance δ and rate $\dot{\delta}$ needed to generate F_n , and in turn $\mathbf{F}_C$ and $\mathbf{M}_C$ can be computed. For simplicity, we assume the wall has a known location d along the X axis, and spans the YZ plane. This solution can be modified for a flat contact surface spanning any plane. Pertinent elements of the contact model are illustrated in Figure 3. First, we parametrize a generic point on the bumper circle in $\mathcal{F}_I$ with:

$$\mathbf{p}_b = R_b \cos \beta \hat{\mathbf{u}} + R_b \sin \beta \hat{\mathbf{n}}^\times \hat{\mathbf{u}} + \mathbf{p}_T \quad (10)$$

where $\hat{\mathbf{n}}$ and $\hat{\mathbf{u}}$ are unit vectors normal and tangent to the circle plane respectively, with both vectors derived using the bumper tilt α indicated in Figure 2b. The circle center $\mathbf{p}_T$ is coincident with the thruster location, and β is the angle from $\hat{\mathbf{u}}$ at which the generic point on the bumper, $\mathbf{p}_b$, is positioned. Using our choice of wall location and orientation, we can solve the following equation for β that

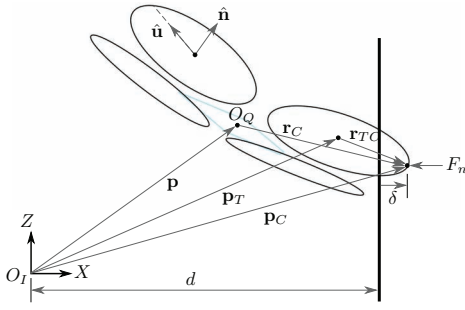


Fig. 3. Contact geometry elements for example impact scenario at a bumper

parametrizes the points of intersection between the contact circle and the wall:

$$d = (R_b \cos \beta \hat{\mathbf{u}} + R_b \sin \beta \hat{\mathbf{n}} \times \hat{\mathbf{u}} + \mathbf{p}_T)^T \mathbf{e}_X \quad (11)$$

We are interested in solutions of (11) that produce two real angles, which indicate a penetration, that is, $\delta > 0$. These solutions are then used in (10) to find the two possible points of intersection. The vector $\mathbf{r}_{TC}$ that bisects the two vectors from the circle center to the points of intersection has a magnitude R_b , and locates the contact point relative to the circle center.

With $\mathbf{r}_{TC}$, we find the position of the contact point $\mathbf{p}_C$, and the relative position of the contact point from the CM $\mathbf{r}_C$:

$$\mathbf{p}_C = \pm \mathbf{r}_{TC} + \mathbf{p}_T \quad (12)$$

$$\mathbf{r}_C = \mathbf{R}(\mathbf{p}_C - \mathbf{p}) \quad (13)$$

where the sign in the first term of (12) will be negative if more than half the circle is enclosed by the wall, and positive otherwise. Based on our known wall location d along the X axis, δ is simply the projection of $\mathbf{p}_C$ onto the X axis subtracted by d :

$$\delta = \mathbf{p}_C^T \mathbf{e}_X - d \quad (14)$$

Finally, rigid body kinematics are used to find the contact point velocity in $\mathcal{F}_Q$, which is then projected onto the X axis to find the penetration rate $\dot{\delta}$:

$$\dot{\delta} = (\mathbf{v} + \boldsymbol{\omega} \times \mathbf{r}_C)^T \mathbf{e}_X \quad (15)$$

C. Full contact model

With the penetration distance and rate determined from the contact geometry, as per (14) and (15), eqn. (8) provides the normal contact force F_n , and eqn. (13) provides the contact location, which are then used to find the force and moment due to impact on the quadrotor at bumper j in $\mathcal{F}_Q$:

$$\mathbf{F}_{Cj} = \pm F_n (\mathbf{R} \mathbf{e}_X), \quad j \in \{1, 2, 3, 4\} \quad (16)$$

$$\mathbf{M}_{Cj} = \mathbf{r}_C \times \mathbf{F}_{Cj}, \quad j \in \{1, 2, 3, 4\} \quad (17)$$

The bumper index $j = 1, 2, 3, 4$ prescribes the bumper locations: starboard front, starboard rear, port rear, and port front, respectively. Note, the sign choice in (16) allows for the quadrotor to approach the wall from either side.

The sum of the forces and moments from the contacting bumpers in (16) and (17) comprise the total contact force and

moment needed for the equations of motion in (1) and (2). Note that the contact geometry only allows for one contact point per bumper, so there can be a maximum of four and a minimum of zero contact points at any given time.

IV. SIMULATION RESULTS

Through an investigation of the quadrotor response predicted by our contact dynamics model for a range of initial impact conditions, we test the model by scrutinizing the trends observed from simulation, and verify that the results agree with our intuition and can be explained in relation to the physical system. Then, a particular impact situation is used to illustrate the full quadrotor post-impact response.

In the following, the position and impact velocity of the vehicle refer to those of the quadrotor CM. For a more intuitive understanding of the results, the Tait-Bryan Euler angles $\{\phi, \theta, \psi\}$, denoting the roll, pitch, and yaw are presented instead of the quaternion parametrization they are derived from.

A. Simulation setup

The dynamics model in Section II with contact forces derived from Section III was simulated using *MATLAB*, with the trajectory controller running at 200 Hz, the same speed as the on-board controller of our experimental platform, and the equations of motion simulated with *ode45* at variable integration time steps (> 200 Hz).

Spiri's inertial, geometric and contact parameters are provided in Table I. Due to the complex structure and uncertainty in material composition of our experimental platform's bumpers, the contact parameters k and n in (8) were determined experimentally by measuring the bumper deflection under different static compressive loads at the point on the bumper's outside edge, farthest from its location of attachment to the body. The coefficient of restitution e for use in (9) was chosen to reflect a nearly elastic collision, which is reasonable given the flexibility of the bumpers, and was set to be constant since variations in its value had negligible effect on the simulated response. Even though the contact parameters are likely to vary with incoming impact orientations and speeds [27], the values used here are reasonable for a first approach at capturing the contact mechanics.

Direction of the quadrotor into the wall at a specified angle and impact velocity was achieved by prescribing a desired attitude set-point to the attitude controller, and specifying the initial position and velocity of the quadrotor. The attitude controller is based on a PID law for ϕ and θ , and a PI controller for the yaw rate r . A double-loop PID altitude controller [30] maintains the desired height at 2 m for the duration of the simulation.

The response of the quadrotor after the first impact with the wall is examined using a nominal control strategy which simply continues using the pre-impact control law and reference inputs, and would represent a scenario where the vehicle is unaware of the collision.

TABLE I
QUADROTOR PARAMETERS

Inertial and Geometric Parameters				
m (kg)	0.933			
I (kg · m ²)	$\begin{bmatrix} 8.737 \times 10^{-3} & -4.20 \times 10^{-7} & -5.29 \times 10^{-5} \\ -4.20 \times 10^{-7} & -8.988 \times 10^{-3} & -1.14 \times 10^{-6} \\ -5.29 \times 10^{-5} & -1.14 \times 10^{-6} & 1.714 \times 10^{-2} \end{bmatrix}$			
$\mathbf{r}_{Tj}$ (m)	$j = 1$	$j = 2$	$j = 3$	$j = 4$
	$\begin{bmatrix} 0.1319 \\ 0.1265 \\ -0.0242 \end{bmatrix}$	$\begin{bmatrix} -0.1212 \\ 0.1265 \\ -0.0242 \end{bmatrix}$	$\begin{bmatrix} -0.1212 \\ -0.1265 \\ -0.0242 \end{bmatrix}$	$\begin{bmatrix} 0.1319 \\ -0.1265 \\ -0.0242 \end{bmatrix}$
Propeller Parameters				
J_r (kg · m ²)	2.207×10^{-5}			
k_t	7.015×10^{-8}			
d_t	9.610×10^{-10}			
Contact Parameters				
k (N/m)	40.0	R_b (m)	0.11	
n	0.54	α (°)	11.0	
e	0.90	d (m)	1.50	

B. Impact responses vs. initial conditions

The quadrotor post-impact response was examined for a range of ‘inclination’ angles from -20° to 20° , this angle measured between the projection of the z body axis onto the XZ inertial plane and the Z inertial axis, with a positive value when the quadrotor is tilted towards the wall. For example, when $\psi = 0^\circ$, the inclination angle is simply $-\theta$. The effect of inclination on the maximum deflection during the initial impact, δ_{init} , is examined for three impact speeds in the X direction, $\dot{X}_C$, for a two bumper initial impact ($\psi = 0^\circ$) in Figure 4a and a one bumper initial impact ($\psi = 45^\circ$) in Figure 4b. The impact for the $\psi = 0^\circ$ case occurs at bumpers 1 and 4 and δ_{init} is taken as the maximum deflection for the two bumpers.

The peak deflection values occurring consistently at an inclination of $\sim 10^\circ$ for all curves in Figure 4 correspond to the colliding bumper circles oriented near horizontally (recall the bumper tilt angle relative to the body is $\alpha = 11^\circ$), as contacts at this inclination angle will result in negligible contact moment $\mathbf{M}_C$ applied to the vehicle, and hence, a more direct impact resulting in larger δ_{init} . The deflection curves are also asymmetric about the peak values which can be explained by the location of the contact point relative to the center of mass: $\mathbf{p}_C$ is above the CM for inclinations less than $\sim 10^\circ$, and below the CM otherwise. In agreement with [31], when $\mathbf{p}_C$ is above the CM, $\mathbf{M}_C$ rotates the quadrotor away from the wall, which results in a smaller δ_{init} than when $\mathbf{M}_C$ rotates the quadrotor toward the wall. Also of note are the higher rates of change of δ_{init} predicted for negative inclination angles, since in these cases, the quadrotor is decelerating at the moment of impact to achieve the desired $\dot{X}_C$, and therefore, has less inertia towards the wall.

Figure 4 also shows higher deflections δ_{init} for increasing

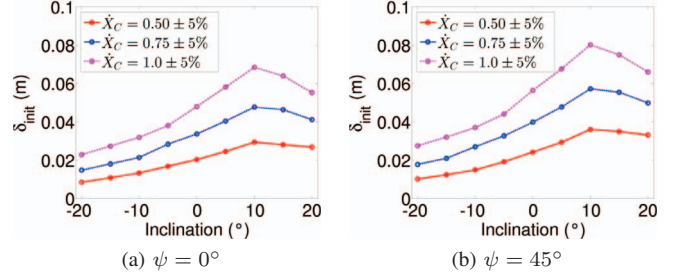


Fig. 4. Maximum bumper deflection at initial impact for different initial conditions

TABLE II
POST FIRST IMPACT SCENARIOS

ψ (°)	$\dot{X}_C$ (m/s)	Inclination (°)								
		-20	-15	-10	-5	0	5	10	15	20
0	1.00	B	B	B	B	B	B	C/7	D/6	D/7
	0.75	B	B	B	B	B	A	C/9	D/7	D/7
	0.50	B	A	A	A	A	A	C/8	C/12	D/6
45	1.00	B	B	B	B	B	B	D/7	D/5	D/7
	0.75	B	B	B	B	B	A	C/7	D/7	D/7
	0.50	A	A	A	A	A	A	C/6	D/5	D/7

$\dot{X}_C$, as expected. Slightly lower δ_{init} are observed for the two bumper initial impact case than the one bumper case—also expected as there are two contact forces resisting deflection in the former case, as opposed to only one in the latter. Table II summarizes the four post initial impact response types generally observed for the initial impact conditions studied. Note that a ‘crash’ occurs in simulation when the vehicle reaches $Z = 0$ m. The quadrotor of type:

- A) bounces away from the wall, maintains stability and does not crash.
- B) is destabilized by the initial impact into a high-pitch orientation or into a somersault-like response and crashes away from the wall.
- C) has an additional impact with the wall on the leading bumper(s), then follows type D) response.
- D) flips towards the wall, has a subsequent impact on the aft bumper(s), then a series of successive impacts with the wall, while ‘sliding’ downwards until crashing.

The responses of type ‘A’ and ‘B’ occur at inclinations below 10° for the reasons described earlier, with type ‘A’ dominating at lower $\dot{X}_C$. For types ‘C’ and ‘D’, the number of subsequent impacts following the initial impact are also indicated in Table II. These two types are differentiated because it is expected that recovery from ‘C’ will be less challenging than from ‘D’.

C. Quadrotor post-impact response

We now examine the simulated response of Spiri for an example scenario using the nominal control strategy described previously in Section IV-A. The desired roll ϕ_{des} , pitch θ_{des} , and yaw ψ_{des} are set to be constant and equal to the corresponding initial angles that achieve the prescribed

inclination and heading. The bumper deflections in Figure 5 clearly show a type ‘D’ response through the alternating deflections of the leading bumper 4 and the aft bumper 2, with minimal deflections at bumpers 1 and 3.

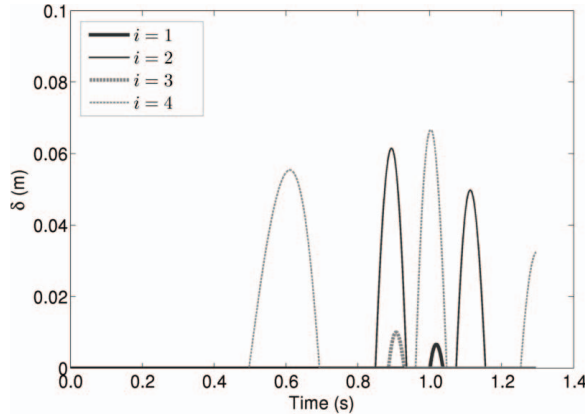


Fig. 5. Bumper deflections for a wall impact at 0.5 s, with $\psi = 45^\circ$, $\dot{X}_C = 0.75$ m/s, and Inclination = 15° .

The vehicle states are displayed in Figure 6 (first impact occurs at 0.5 s), and they correspond to a type ‘D’ response described above. During the first ~ 0.3 s following impact, the quadrotor is flipping towards the wall, which maintains the quadrotor CM at a relatively level altitude, after which the altitude decreases significantly while the quadrotor ‘slides’ downwards. The X position fluctuates around the wall location d , which occurs during the successive impacts on the aft and leading bumpers, and the Y position is stable, due to the minimal impacts at the side bumpers 1 and 3.

The quadrotor post-impact attitude response shows clearly that the nominal attitude controller cannot compensate for the impact force and moment, as the desired Euler angles cannot be tracked. Rotation during the successive impacts at the aft and leading bumpers occurs primarily around the ϕ and ψ axes. The local extrema of the attitude response coincide with the instances of peak bumper deflections in Figure 5.

V. EXPERIMENTAL VALIDATION

A. Experimental setup

Twelve live experiments were performed on Spiri quadrotor flown into a wall, under manual joystick attitude and thrust inputs, at varying initial pitch angles and incoming velocities. The attitude controller used in the experiments was of the same form as the controller used in simulations, as described in Section IV; recovery control has not been attempted to date and will be the focus of future work. In every experiment, the quadrotor was destabilized by the collision with the wall into an aerobatic motion, followed by crashing, undamaged, onto a foam crash bed.

Data sets from the on-board IMU and Vicon motion-capture system were logged using ROS at approximately 20 Hz and 60 Hz, respectively. These data were post-processed to determine the vehicle’s pre-impact orientation and velocity. Additionally, the motor speeds were logged at

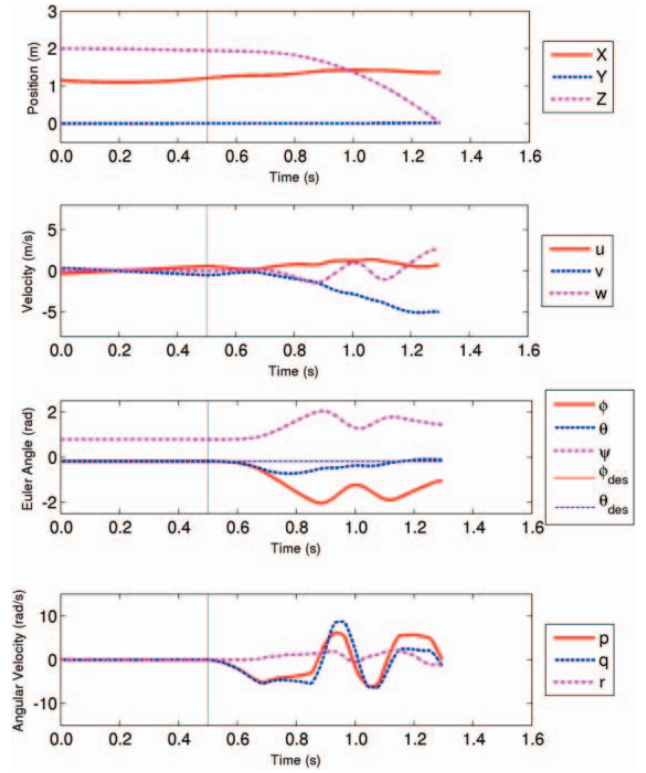


Fig. 6. Quadrotor simulated response for a wall impact at 0.5 s, with $\psi = 45^\circ$, $\dot{X}_C = 0.75$ m/s, and Inclination = 15° .

approximately 20 Hz, in order to simulate the post-impact responses for comparison to the experiment.

B. Reconstruction of collision conditions in simulation

To demonstrate correspondence between the responses simulated with the developed dynamics model, as presented in Sections II and III, and the live crash tests, seven out of twelve experimental tests with viable recorded data were reconstructed in simulation. Careful inspection of the video footage as well as the motor speeds post-impact data revealed that motor controller safety power-off was triggered by the motor stall when a propeller/bumper experienced impact with the wall. Accordingly, generation of simulated responses for comparison to experimental results required matching not only the respective initial conditions of the vehicle, but also the post-impact motor speeds.

The impact event was identified from the spikes in the accelerometer readings. The attitude of Spiri at the instant of impact was computed as a weighted average of the IMU filtered attitude, the attitude estimate from the motion-capture system, and the angle between Spiri’s xy body plane and the wall, as estimated from a video frame at impact. The incoming horizontal and vertical velocities were estimated using a band pass filter on the motion-captured position data with cut-off frequencies at 10 and 50 Hz. The simulated initial impact conditions all lie within the error ranges of the estimated initial conditions in Table III, which in turn were generated according to the discrepancies seen between the different measurement methods.

TABLE III
EXPERIMENTAL IMPACT CONDITIONS AND RESPONSE TYPE
COMPARISON

Crash	Experiment				Simulation
	ψ (°) ± 6.4	$\dot{X}_C$ (m/s) ± 0.01	Inclination (°)	Response Type	Response Type
1 *	20	0.7	4.5 ± 7	B	B
2	10	1.1	4.4 ± 7	B	B
3	14	1.1	6.8 ± 7	B	B
4	14	1.3	11 ± 8	B	B
5	11	2.0	15 ± 7	Other	Other
6	4.0	2.7	16 ± 7	D	D
7	11	2.0	19 ± 8	D	D

*In this crash, although the simulated and experimental response types match, there is no visual correspondence between post-impact experimental and simulated behaviours. A discussion follows in the text.

As a result of motor power-off on impact noted earlier, the recorded motor speed data showed that the four motor speeds decreased at varying rates post-impact towards zero. This was imitated in simulation by decreasing the four propeller angular velocities at matching rates to those recorded from experiment.

C. Comparison to simulated response

Table III summarizes the initial impact conditions measured during the seven viable experiments and compares the post-impact response types seen in experiment versus simulation. The first observation on these results is that only type ‘B’ and ‘D’ responses of the vehicle occurred: this is as a consequence of the motor power-off phenomenon since both of these response types involve the vehicle flipping towards the wall after the first impact—a behaviour consistent with loss of thrust on the impacted motors. Secondly, with the exception of Crash 5, the experimental response types were matched in simulation. Furthermore, good correspondence was observed in visual comparison of simulated and experimental post-impact behaviour for the experiments with matching response types, except for Crash 1. Example side-by-side comparisons of snapshots from the experimental and simulated collisions are seen in Figure 7 for Crash 3 (type ‘B’), and Figure 8 for Crash 6 (type ‘D’).

An examination of the experimental collision video and the recorded motor speeds revealed a plausible explanation for the lack of visual correspondence for Crash 1. Upon impact, in addition to the leading bumper deforming, the corresponding propeller tip also deformed significantly: this phenomenon did not occur in other crashes nor is it captured by our contact dynamics model. As well, the recorded motor speeds show the motor associated with the leading bumper powered-off last, in contrast to the other crashes where the leading motor powers-off first. We speculate that the impact on the propeller tip caused the self-tightening propeller to disengage from the motor, allowing the motor to continue its operation while generating no thrust. Therefore, using the recorded motor speeds as control inputs in (5) to (7)

would not generate the proper thruster forces and moments in simulation.

As indicated in Table III, the response exhibited in Crash 5 does not fall into any one of the categories outlined in Section IV-B. In experiment, the first impact turned the quadrotor sideways while it flipped towards the wall, leading to a second impact on only one of its aft bumpers, where significant deformation was seen in the direction *transverse* to the propeller plane. A somersault-like response *away* from the wall followed, and the aircraft crashed without additional impacts. In simulation, after the second impact on the single aft bumper, the aircraft continued its flipping motion *towards* the wall, and then crashed away from the wall without additional impacts. Our contact model does not capture properly the transverse stiffness of the bumpers, nor the additional flexibility and play in the transverse direction caused by the bumper attachment method to the body. Hence, a crash resulting in large impact forces directed transversely to the rotor plane will not be accurately simulated with the present model.

VI. CONCLUSION AND FUTURE WORK

We have developed a model for a quadrotor with protective bumpers that captures its dynamic response due to a non-destructive collision with a stationary, vertical wall. The standard quadrotor dynamics model was augmented with a non-linear compliant contact force model, using contact geometry derived from a representative model of the bumpers. The model was validated in simulation, showing logical responses under different initial impact conditions. From this array of simulations, the post-impact quadrotor response has been generally categorized for use in future work; ultimately, we speculate the controller and/or parameters will need to be varied for recovery from post-impact responses. The experimental testing affirms the notion of ‘non-destructive’ collisions and, hence, viability of recovery. From the seven experiments with viable data to recreate the experimental conditions in simulation, qualitative correspondence between the experimental and simulated responses was observed in five of the tests, and logical hypotheses for lack thereof in the remaining experiments were put forward. This correspondence shows that the proposed contact model is able to capture real life impact scenarios between a quadrotor and a wall for the purposes of developing a post-impact recovery controller.

The experiments also uncovered that other considerations are important for full control of a quadrotor post-impact, besides the controller itself, which we will address in future work. In particular, in the initial stages of the recovery controller development, optimal operation of the motors is needed and hence, the motor power-off observed in our experiments must be prevented. In the short term, we will address this problem by redesigning the bumpers of Spiri vehicle to reduce their flexibility, thus ensuring they do not deform to the point of causing motor stall. Furthermore, a reduction in their pliability in the transverse direction to the propeller plane will increase the model’s accuracy, allowing

us to better predict the quadrotor response over the range of impact conditions. With the high fidelity contact model, the recovery controller will be designed and evaluated in simulation to determine if and under what initial conditions recovery is possible, which will be the final step before full development of a successful live impact recovery controller.

ACKNOWLEDGEMENTS

This work was supported by the National Sciences and Engineering Research Council (NSERC) Canadian Field Robotics Network (NCFRN) and the McGill Engineering Undergraduate Student Masters Award (MEUSMA). The authors would like to thank Khoi Tran and Thomas Fuehrer for their assistance on performing the experimental tests. We would also like to thank Pleiades Robotics, Inc. for their technical support on the Spiri quadrotor platform.

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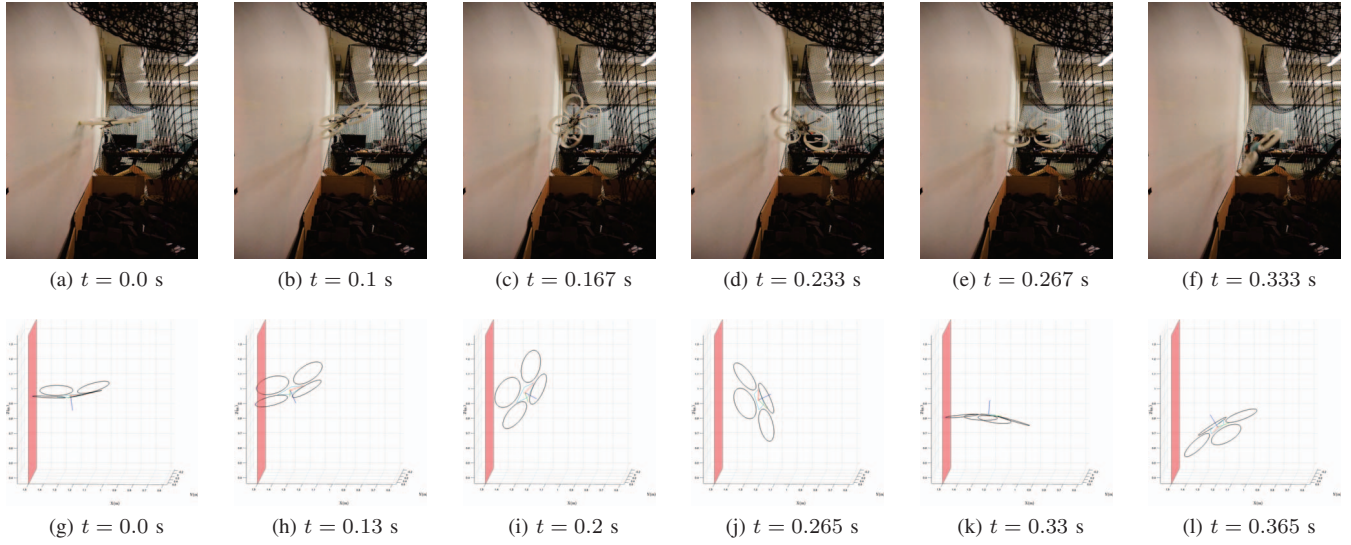


Fig. 7. Qualitative comparison of experimental (a-f) and simulated (g-l) quadrotor response with matching initial impact conditions for Crash 3, exhibiting a Type ‘B’ response. Time after impact is indicated in the subfigure captions. Images (a) and (g) mimic the initial impact conditions, (b) and (h) show the initial destabilization of the quadrotor, and (c-f) and (i-l) show the somersault-like response of a type ‘B’ collision, which leads to a crash away from the wall.

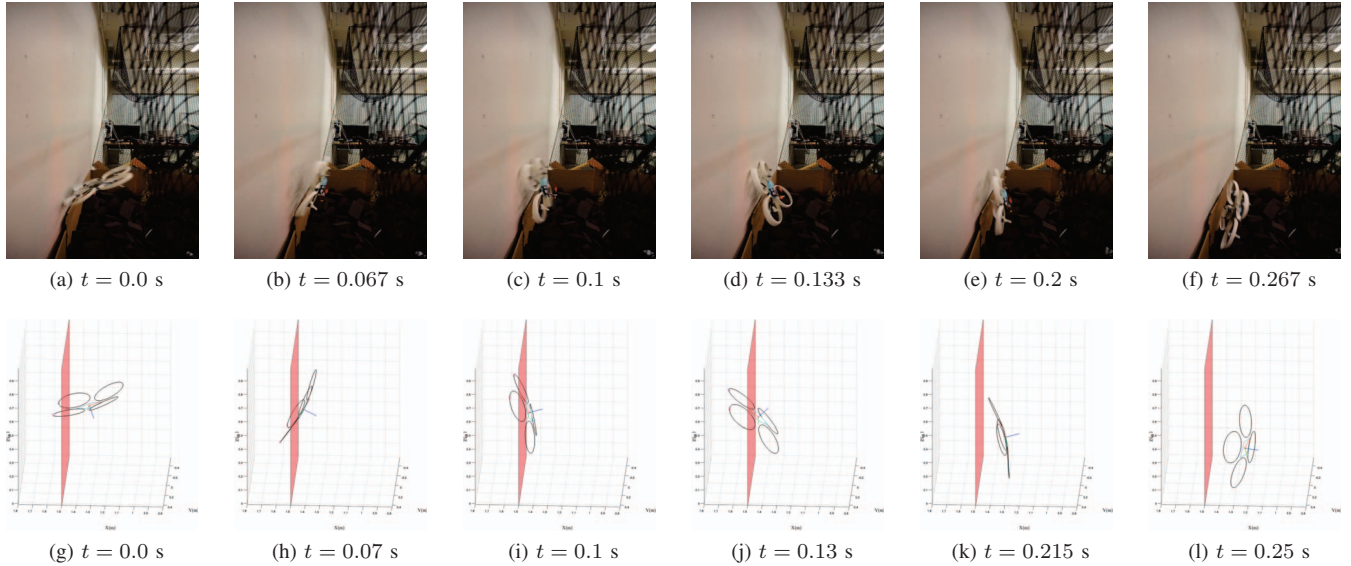


Fig. 8. Qualitative comparison of experimental (a-f) and simulated (g-l) quadrotor response with matching initial impact conditions for Crash 6, exhibiting a Type ‘D’ response. Time after impact is indicated in the subfigure captions. Images (a) and (f) mimic the initial impact conditions and initial deformation, (b) and (h) show the pitching into the wall of a type ‘D’ collision, (c-d) and (i-j) show destabilization of the quadrotor and the impact of the aft bumpers after pitching into the ‘upside-down’ range, and finally (e) and (k) show the beginning of the quadrotor’s descent to the ground (foam bed in the live test).